Fact

Given a function $f(x)$ if,

$$\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = L$$

then the normal limit will exist and

$$\lim_{x\to a} f(x) = L$$

Likewise, if

$$\lim_{x\to a} f(x) = L$$

then,

$$\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = L$$

This fact can be turned around to also say that if the two one-sided limits have different values, *i.e.*,

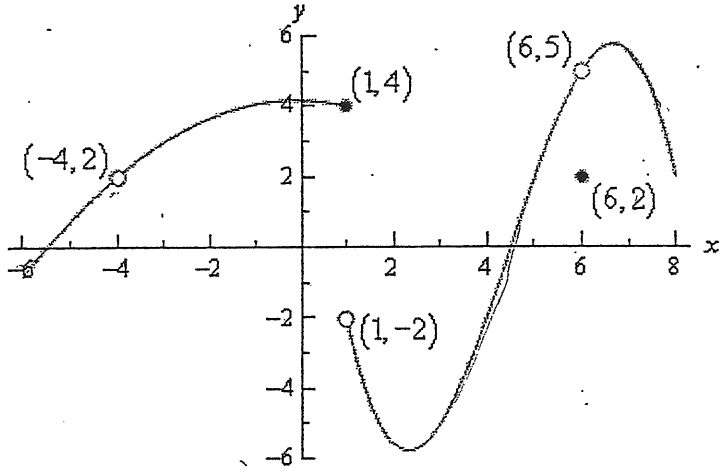
$$\lim_{x\to a^+} f(x) \neq \lim_{x\to a^-} f(x)$$

then the normal limit will not exist.

This should make some sense. If the normal limit did exist then by the fact the two one-sided limits would have to exist and have the same value by the above fact. So, if the two one-sided limits have different values (or don't even exist) then the normal limit simply can't exist.

Let's take a look at one more example to make sure that we've got all the ideas about limits down that we've looked at in the last couple of sections.

Example 4 Given the following graph,



compute each of the following.

(a) $f(-4)$ (b) $\lim_{x\to -4^-} f(x)$ (c) $\lim_{x\to -4^+} f(x)$ (d) $\lim_{x\to -4} f(x)$

(e) $f(1)$ (f) $\lim_{x\to 1^-} f(x)$ (g) $\lim_{x\to 1^+} f(x)$ (h) $\lim_{x\to 1} f(x)$

(i) $f(6)$ (j) $\lim_{x\to 6^-} f(x)$ (k) $\lim_{x\to 6^+} f(x)$ (l) $\lim_{x\to 6} f(x)$

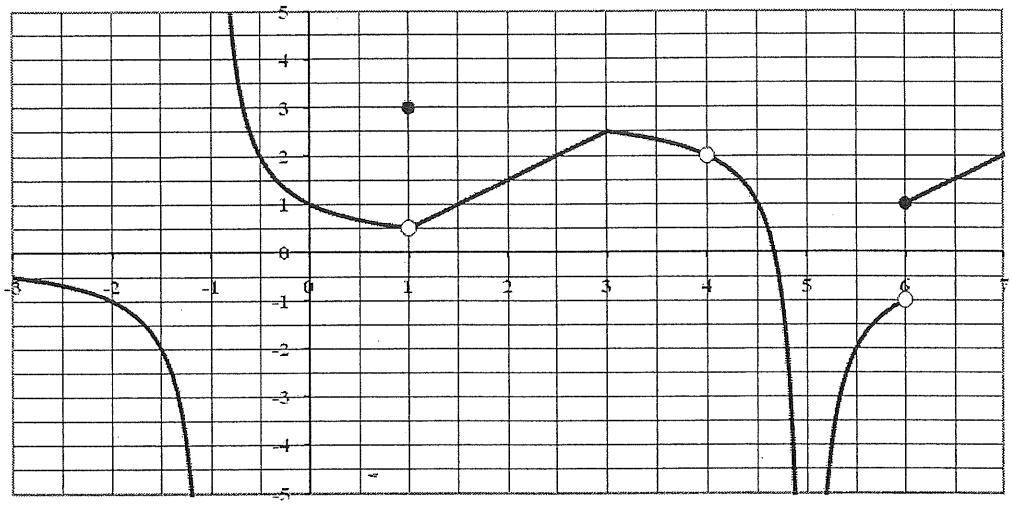
Name _____

Precalculus Limit Notes

Evaluating Limits Graphically and Numerically

I. Evaluating Limits Graphically

1. The function f has the graph shown. Using this graph, fill out the table below. If the limit does not exist, state this.



a	$f(a)$	$\lim_{x\to a^-} f(x)$	$\lim_{x\to a^+} f(x)$	$\lim_{x\to a} f(x)$
-2				
-1				
0				
1				
4				
5				
6				

II. Evaluating a limit numerically.

2(a) Let $f(x) = \frac{(x-1)(x+2)}{(x-3)}$ Use your calculator to fill in each chart and then evaluate each one-sided limit below.

x	2.9	2.99	2.999	2.9999
f(x)				

x	3.1	3.01	3.001	3.0001
f(x)				

$\lim_{x\to 3^-} f(x) =$ $\lim_{x\to 3^+} f(x)$

(b) Does $\lim_{x\to 3} f(x)$ exist? Why or why not?